APPLICATIONS OF MULTIRESOLUTION BASED FDTD MULTIGRID

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Abstract- A Multigrid 2-D Finite Difference Time Domain (FDTD) technique based on Multiresolution analysis with Haar wavelets is used to analyze structures such as an empty waveguide and a shielded stripline. The results obtained are compared with those computed using a finer resolution regular FDTD mesh. This comparative study illustrates the benefits of using wavelets in FDTD analysis.

I Introduction

Multiresolution Time Domain (MRTD) Technique is a new approach to solving time domain problems. This technique uses Multiresolution Analysis (MRA) to Discretize Maxwell's equations in time domain and demonstrates excellent capability in solving Electromagnetics problems [1], [2]. Depending on the choice of basis functions, several different schemes result, each one carrying the signature of the basis functions used in MRA. It is also important to note that the design of an MRTD scheme can be accomplished using one's own application-specific basis functions. MRTD technique using Haar scaling functions results in the FDTD technique [3].

Recently, an FDTD multigrid using the Haar wavelet basis has been developed and it has been demonstrated that such a scheme exhibits highly linear dispersion characteristics [3]. Motivation for this work stems from the theory of MRA which says that a function which is expanded in terms of scaling functions of a lower resolution level, m1, can be improved to a higher resolution level, m2, by using wavelets of the intermediate levels. In other words, expanding a function using scaling function of resolution level m1 and wavelets up to resolution level m2 gives the same accuracy as expanding the function using just the scaling functions of resolution m2. However, the use of wavelet expansions has major implications in memory savings due to the fact that the wavelet expansion coefficients are significant only in areas of rapid field variations. This allows for the capability to discard wavelet expansion coefficients where they are not significant thereby leading to significant economy in memory. Different resolutions of wavelets can be combined so as to locally improve the accuracy of the approximation of the unknown function. This, combined with the fact that wavelet coefficients are significant only at abrupt field variations and discontinuities allows MRTD to lend itself very naturally to a Multigrid capability.

In this paper, a 2D MRTD scheme based on Haar basis functions (first order resolution) is developed and applied to solve for the Electromagnetic fields in a

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waveguide and a shielded stripline. The results obtained are compared with those computed using conventional FDTD technique. It will be shown that the wavelet coefficients are significant only at locations with abrupt field variations. This facilitates in obtaining accurate solutions by combining the wavelet and scaling coefficients only in regions where the wavelet coefficients are significant (discontinuities).

II The 2D-MRTD scheme

Consider the following 2-D scalar equation obtained from Maxwell's H-curl equation:

$$\epsilon\frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} + \beta H_y \qquad (1)$$

This equation can be rewritten in a differential operator form as shown below:

$$L_1(f_1(x,y,t)) + L_2(f_2(x,y,t)) = g \qquad (2)$$

where L_1 and L_2 are the operators and f_1(x,y,t) and f_2(x,y,t) represent the electric/magnetic fields. We now expand the fields using a Haar based MRA with scaling functions ϕ and wavelet functions ψ [3]. The field expansion can be represented as follows:

$$f(x,y,t) = [A][\phi(x)\phi(y)] + [B][\phi(x)\psi(y)] + [C][\psi(x)\phi(y)] + [D][\psi(x)\psi(y)] \qquad (3)$$

where [ϕ(x)ϕ(y)], [ϕ(x)ψ(y)], [ψ(x)ϕ(y)] and [ψ(x)ψ(y)] represent matrices whose elements are the corresponding basis functions in the computation domain of interest and [A], [B], [C], [D] represent the matrices of the unknown coefficients which give information about the fields and their derivatives.

Application of Galerkin's technique leads to four schemes which can be represented as follows:

$$< [\phi\phi], L_1(f_1) + L_2(f_2) >=< [\phi\phi], g >: \phi\phi Scheme$$
$$< [\phi\psi], L_1(f_1) + L_2(f_2) >=< [\phi\psi], g >: \phi\psi Scheme$$
$$< [\psi\phi], L_1(f_1) + L_2(f_2) >=< [\psi\phi], g >: \psi\phi Scheme$$
$$< [\psi\psi], L_1(f_1) + L_2(f_2) >=< [\psi\psi], g >: \psi\psi Scheme$$

From this system, we obtain a set of simultaneous discretized equations. For the first resolution level of Haar wavelets, the above four schemes decouple and coupling can be achieved only through the excitation term and the boundaries.

The shielded structures analyzed here are terminated at Perfect Electric Conductors (PEC) and the boundary conditions are obtained by applying the natural boundary condition for the electric field on a PEC as shown below:

$$E_t^{\phi\phi}\phi(x)\phi(y) + E_t^{\phi\psi}\phi(x)\psi(y) + E_t^{\psi\phi}\psi(x)\phi(y) + + E_t^{\psi\psi}\psi(x)\psi(y) = 0....AtPEC. \qquad (4)$$

where $E_t^{\phi\phi}$, $E_t^{\phi\psi}$, $E_t^{\psi\phi}$ and $E_t^{\psi\psi}$ are the scaling and wavelet coefficients of the tangential electric field at the boundary nodes.

The above equations are discretized by the use of Galerkin's method which results in a set of matrix equations of order N = M+1 where M is the order of the considered wavelet resolutions. These equations are solved simultaneously with the discretized Maxwell's equations to numerically apply the correct boundary conditions.

III Applications of 2D FDTD Multigrid and Results

The 2-D MRTD scheme derived above has been applied to analyze the Electromagnetic fields in a waveguide and a shielded stripline.

(a) Waveguide: An empty waveguide with cross-section of 12.7 x 25.4 mm is chosen. A coarse 5 x 8 mesh is used to discretize this mesh and 2D MRTD technique was applied to analyze the fields in this geometry. Fig. 1 shows the amplitudes of the wavelet and scaling coefficients of the electric field obtained by using MRTD technique. From this figure it can be seen that only the $\phi\phi$ and $\phi\psi$ coefficients make a significant contribution to the field and that the contribution of $\psi\phi$ and $\psi\psi$ is negligible. From the computed coefficients, the total field is reconstructed using an appropriate combination of the scaling and significant wavelet coefficients. For the waveguide chosen here, elimination of the wavelet coefficients that have no significant contribution leads to 480 unknowns. The reconstructed field obtained by this mesh has the same accuracy as that of a 10 x 16 FDTD mesh with 960 unknowns which is in agreement with the theory of MRA. Fig. 2 shows the results of this comparison and demonstrates that the use of multigrid scheme provides a 50% economy in memory.

(b) Shielded Stripline : Next, a stripline of width 1.27mm is considered. It is enclosed in a cavity of area 12.7 x 12.7 mm so that the side walls are sufficiently far away to not affect the propagation. The strip is placed 12.7mm from the ground. A 40 x 40 mesh is used to analyze the fields in this geometry with the 2D MRTD technique. Fig. 3 shows the derived scaling and wavelet coefficients of the fields just below the strip. From the figure, it can be seen that among the wavelet coefficients, only $\psi\phi$ makes a significant contribution close to the vicinity of the strip where the field variation is rather abrupt. Fig. 4 shows the comparision of the total reconstructed field in the 40 x 40 MRTD mesh with that of a 40 x 40 and 80 x 80 FDTD mesh. From the figure it is clear that the field computed by 40 x 40 MRTD mesh using only the significant wavelet coefficients follows the results of the finer 80x80 mesh very closely, demonstrating once again the significant economy in memory as illustrated in Table 1. Fig. 5 shows the Normal Electric field plot of the strip and the variable mesh resulting from MRTD.

IV Conclusion

A Haar wavelet based 2D MRTD scheme was developed and applied to analyse the fields in a waveguide and a shielded stripline. The wavelet coefficients obtained are significant only in regions of rapid field variations. Thus the FDTD multigrid capability using MRTD technique has demonstrated significant economy in memory.

Table 1: Comparison of the memory requirements in FDTD and MRTD techniques

Technique	Unknown Coeff.
40x40 FDTD	9600
40x40 MRTD	11328
80x80 FDTD	38400

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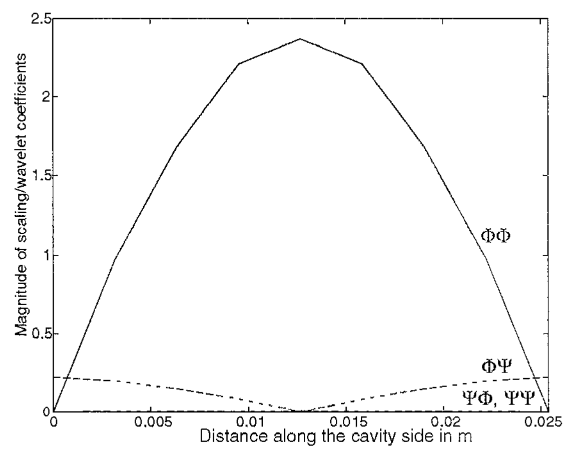

Figure 1: Amplitudes of Scaling and Wavelet Coefficients in a Waveguide.

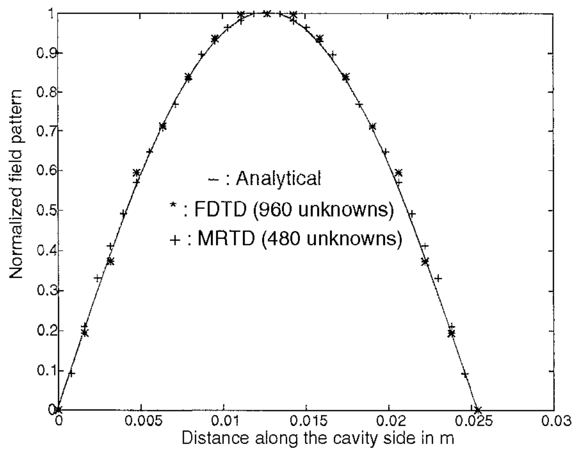

Figure 2: Comparison of MRTD, FDTD and Analytical Fields in a Waveguide.

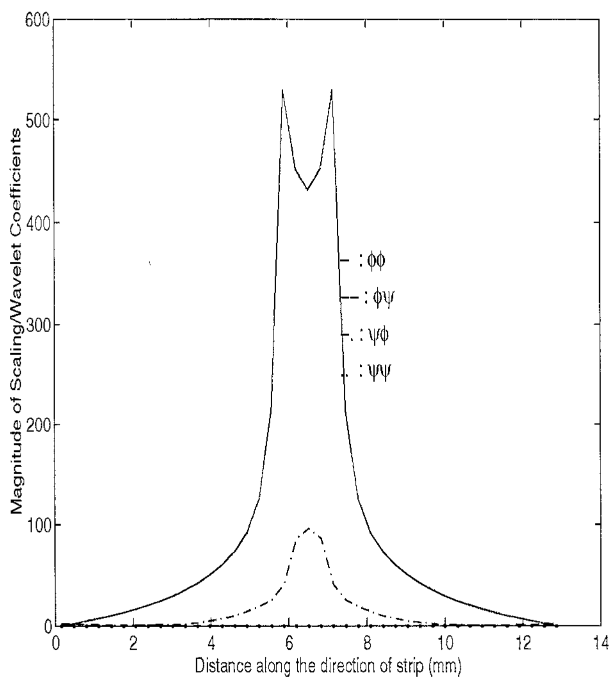

Figure 3: Amplitudes of Scaling and Wavelet Coefficients of a Shielded Stripline.

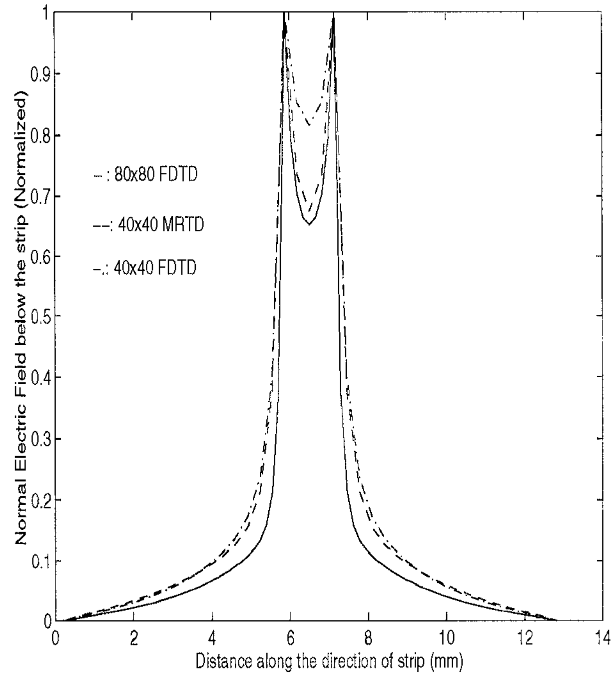

Figure 4: Comparison of Normal Electric Field under a stripline using MRTD and FDTD techniques.

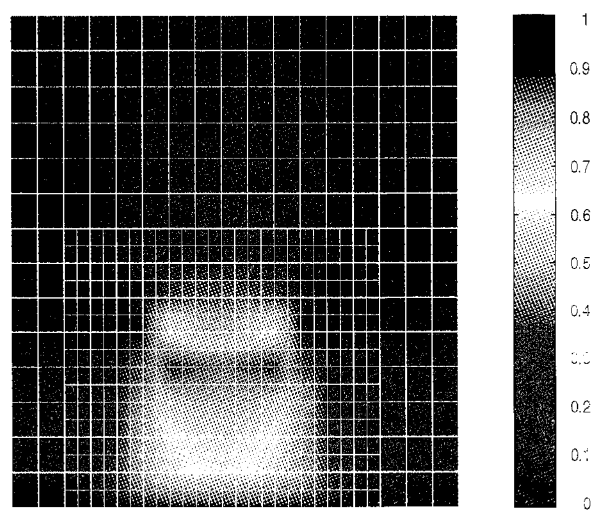

Figure 5: FDTD Multigrid and Field Plot of the Stripline.

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